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MATHEMATICAL DESCRIPTION OF A MARINE GAS TURBINE SYSTEM

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Philadelphia, Pennsylvania

27 October 1972

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OF A
MARINE GAS TURBINE SYSTEM

NAVSECPHILADIV RDT&E Project C-53-II
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27 October 1972

by

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GLOSSARY OF NOTATION

B	Function	lb/hr
C	Function	lb/hr
D	Function	rpm
E	Function	lb/hr
E_c	Chemical Energy	lb ft
E_s	Total System Rotational Energy	lb ft
E_{sp}	Rotational Energy of Power Turbine Assembly	lb ft
e	Reciprocal of Dimensionless Specific Fuel Consumption	-
f	Function	lb ft
g	Function	rpm
h	Function	lb/hr
j	Integer	-
J_h	Net Polar Moment of Inertia of High Speed Turbine-Compressor Pair	lb ft sec^2
J_l	Net Polar Moment of Inertia of Low Speed Turbine-Compressor Pair	lb ft sec^2
J_p	Net Polar Moment of Inertia of Power Turbine and Load	lb ft sec^2
k	Integer	-
L	Throttle Position	%
L_i	Throttle Position at Idle Fuel Rate	%

GLOSSARY OF NOTATION (CONT'D)

L_k	Throttle Position at the Fuel Rate, W_{fk}	%
m	Integer	-
n	Integer	-
N_h	Speed of High Speed Turbine	rpm
N_{hs}	Speed Set Point of High Speed Turbine	rpm
N_l	Speed of Low Speed Turbine	rpm
N_p	Speed of Power Turbine	rpm
Q_a	Applied Torque	lb ft
Q_l	Load Torque	lb ft
Q_p	Power Turbine Torque	lb ft
s	Dimensionless Specific Fuel Consumption	-
t	Time	sec
W_f	Fuel Rate	lb/hr
W_{fi}	Idle Fuel Rate	lb/hr
W_{fk}	Function	lb/hr
W_{fm}	Maximum Fuel Rate	lb/hr
W_{fqs}	Quasi-Static Fuel Rate	lb/hr
ΔW_f	Accelerating Fuel Rate	lb/hr
ΔpQ	Net Power Turbine Torque	lb ft
ΔN_h	Gas Generator Speed Error	rpm
ϵ	Constant	sec
τ_{fm}	Maximum Time Constant of Fuel Controller	sec

ABSTRACT

A mathematical model of a general marine gas turbine system was developed in an effort to provide an analytical representation from which the dynamics equations of specific systems are deducible. The model is comprised of a pair of coupled, non-linear, first-order, ordinary differential equations. Supplemental data is required to effect specialization of these equations to establish representations of specific systems. These data are derivable from the steady-state characteristics of the engine and the dynamic responses of the fuel control system to select input excitations.

SUMMARY PAGE

The Problem

To develop a mathematical representation of a general marine gas turbine system.

Findings

The equations which govern the transient behavior of such a system can be derived from physical principles independent of specific system design information. Consequently, these equations can be applied to a variety of gas turbine engines for purposes of simulation or control system development.

Recommendations

Specialize these equations to obtain a model of the LM-2500 engine. Develop a computer simulation of the model system in preparation for correlation studies with the real system.

ADMINISTRATIVE INFORMATION

Authority for the research described in this report is contained in NAVSECPHILADIV Semi-Annual Program Summary of 1 November 1972. The RDT&E Subproject No. assigned was S-F35-432-105, Task 12504; the NAVSECPHILADIV Project No. is C-53-11.

This is a final report, the second of a series(1) of reports describing studies related to the dynamics of a marine propulsion gas turbine system.

[1] Superscripts refer to references in the Bibliography.

REPORT OF INVESTIGATION

INTRODUCTION

The need for a generalized analytical model of a gas turbine system is generated by the variety of engines being used or proposed for naval propulsion plants. Representations of specific systems are specializations of this model achieved by imposing the requisite mathematical constraints. The resulting equations can be mechanized on a computer to serve as a dynamic test system for research being performed under related projects.

The DIVISION is currently engaged in a number of projects concerned with the dynamic response characteristics of gas turbine systems. Design of a control system for a high speed waterbrake has been undertaken in an effort to simulate the dynamic loading encountered by gas turbines driving controllable pitch propellers. Preliminary design evaluations will be conducted on a computer, and therefore will require a computer synthesis of the plant. This simulation may also be used for developmental testing of a method designed to determine the dynamic characteristics of nonlinear systems. The method is an analytical procedure that requires dynamic response data acquired by linear test methods for the construction of a system representation. The nonlinear characteristics of the gas turbine system will require the application of the full machinery of this analysis to the model transient test data and thereby exhaust possible problem areas in the analysis.

DESCRIPTION OF SYSTEM

The engine is composed of two major units, an axial flow gas generator and a free (or power) turbine. The external casings of the gas generator and free turbine are connected together to form a continuous duct for the passage of gases through the engine. Coupling between the gas generator and power turbine is aerodynamic. The gas generator consists of two multi-stage compressors, a combustion section and two multi-stage reaction turbines. A concentric shaft is used to connect the compressors to their corresponding turbines.

System input energy is supplied by fuel pump and controlled by a multi-input, single-output fuel controller. The controller delivers part of the fuel to the engine and routes the remainder back to the pump interstage. Engine accelerations are initiated at the throttle (one of the several controller inputs) and follow schedules defined by the controller which is programmed to avoid overtemperature and surge limits of the gas generator. The controller also includes a deceleration schedule that provides for blowout free decelerations.

THE GAS TURBINE

The steady-state characteristics of the power turbine define a mathematical relationship which establishes the torque, Q_p, developed by the power turbine as a function, f, of fuel rate, W_f, and power turbine speed, N_p.

$$Q_p = f(W_f,\ N_p) \qquad \text{lb ft} \qquad (1)$$

The coupling between gas generator turbines is aerodynamic rather than mechanical, however, in the steady-state, the fuel rate alone prescribes the speeds of the compressor-turbine pairs so that a fixed relationship exists between these two different speeds. Moreover, the unoccupied volume between the high and low speed turbines is small. Thus, during accelerations, the speeds of the compressor-turbine pairs tend to change in phase in accordance with their steady-state correspondence. Let N_l and N_h be the respective speeds of the low and high speed turbines. Then, during steady-state or transient conditions, the current speed, N_h determines the speed N_l in the manner prescribed by the following relationship.

$$N_l = g(N_h) \qquad \text{rpm} \qquad (2)$$

Suppose the gas generator system resides at a state characterized by the fuel rate, W_{fo} and the power turbine speed, N_{po}. At the instant, $t = 0$, the fuel rate is instantaneously changed to $W_{fo} + \Delta W_f$. After the time interval, dt, this change in input energy gives rise to an increase in the rotational energy of the entire system. If dN_l, dN_h, and dN_p are

the accompanying speed changes in the low speed, high speed, and power turbine respectively, then the total stored energy increase dE_s, achieved in the interval, dt, is given by the following equation:

$$dE_s = 0.011\ (J_l\ N_l\ dN_l + J_h N_h dN_h + J_p N_p dN_p) \qquad \text{lb ft} \tag{3}$$

The constants J_l, J_h, J_p are the polar moments of inertia of the three mechanically independent rotating assemblies. Equation (2) can be used to eliminate N_l and dN_l from equation (3).

$$dE_s = 0.011\ J_p N_p dN_p + 0.011\ (J_h N_h + J_l\ g(N_h))\ dN \qquad \text{lb ft} \tag{4}$$

Assume that gas generator speed, N_h, and fuel rate have a one-to-one correspondence, h, in the steady-state.

$$W_f = h(N_h) \qquad \text{lb/hr} \tag{5}$$

Let the specific heat release of the fuel be γ. Then the dimensionless specific fuel consumption, s, is given by the equation:

$$s = 2.07\ \frac{\gamma W_f}{N_p Q_p} \tag{6}$$

Eliminate W_f in equation (6) by means of the relationship implied by equations (5) and (1) to obtain s as a function of N_p and N_h (γ is a constant).

$$s = \frac{2.07\,\gamma\ h(N_h)}{N_p Q_p(h(N_h), N_p)} \tag{7}$$

The specific fuel consumption is the ratio of the chemical energy input to the mechanical energy output and as such implies that storage of an additional $J_p N_p dN_p$ units of rotational energy requires the release of $s J_p N_p dN_p$ units of chemical energy. Define e by the equation

$$e = \frac{1}{r} \tag{8}$$

and regard e as a dimensionless variable efficiency. Then the chemical energy increase, dE_c, required to achieve a rotational energy change, dE_s, is given by the following equation:

$$dE_c = dE_s + \left(\frac{1 - e}{e}\right) dE_{sp} \qquad \text{lb ft} \tag{9}$$

where dE_{sp} is the additional energy stored in the power turbine during the interval, dt. Eliminate dE_{sp} and dE_s from equation (9) and substitute the quantity, $0.2168 \gamma \Delta W_f$ dt for dE_c. Divide the resulting equation by dt to determine that:

$$19.7 \qquad W_f = \frac{1}{e} J_p N_p \dot{N}_p + (J_h N_h + J_1 g'(N_h))\dot{N}_h \qquad \text{lb ft} \tag{10}$$

The terms, ΔW_f is the difference fuel rate defined by the equation:

$$\Delta W_f(t) = W_f(t) - W_{fqs}(t) \qquad \text{lb/hr} \tag{11}$$

where $W_f(t)$ is the actual time-dependent fuel rate and W_{fqs} is the quasi-static fuel rate determined by the following procedure. Solve equation (1) for W_f and obtain:

$$W_f = C\ (Q_p,\ N_p) \qquad \text{lb/hr} \qquad (12)$$

Equate Q_p to Q_1 (Q_1 is the load torque) and W_f to W_{fqs} and obtain the result:

$$W_{fqs} = C\ (Q_1,\ N_p) \qquad \text{lb/hr} \qquad (13)$$

Finally, substitute equation (13) into equation (11) and replace ΔW_f in equation (10) with this result.

$$19.7\gamma\ (W_f(t) - C\ (Q_1,\ N_p)) =$$

$$\frac{1}{e} J_p N_p \dot{N}_p + (J_h N_h + J_1 g'(N_h))\ \dot{N}_h \qquad \text{lb ft} \qquad (14)$$

The acceleration, $\dot{N}_p$ of the power turbine is inversely proportional to the total polar moment of inertia (consisting of the power turbine and load) and directly proportional to the net torque, ΔpQ acting on the turbine shaft.

$$\dot{N}_p = \frac{9.54}{J_p}\ \Delta pQ \qquad \text{rpm/sec} \qquad (15)$$

Moreover, the net torque is comprised of the applied torque, Q_a and the load reaction torque, Q_1.

$$\Delta pQ = Q_a - Q_1 \qquad \text{lb ft} \qquad (16)$$

Substitute equation (5) into equation (1) and thereby eliminate W_f from equation (1) and equate this result to Q_a to obtain the following equation.

$$\dot{N}_p = \frac{9.54}{J_p}\ (f\ (h(N_h),\ N_p) - Q_1) \qquad \text{rpm/sec} \qquad (17)$$

Equations (14) and (17) are a system of two coupled, nonlinear, first-order, ordinary differential equations, and hence require two initial conditions: N_p (0) and N_h (0) to uniquely determine the solution. Consequently, the time-dependent behavior of a gas turbine engine is deducible from the solution of the following initial-value-problem (ivp).

$$\dot{N}_p = \frac{9.54}{J_p} \left(f\left(h(N_h), N_p\right) - Q_1 \right) \qquad \text{rpm/sec}$$

$$\frac{1}{e} J_p N_p \dot{N}_p + \left(J_h N_h + J_1 g'(N_h) \right) \dot{N}_h =$$

$$19.7\,\gamma \left(W_f(t) - C(Q_1, N_p) \right) \qquad \text{lb ft} \qquad (18)$$

$$N_p(0) = N_{po} \qquad \text{rpm}$$

$$N_h(0) = N_{ho} \qquad \text{rpm}$$

The derivation of the initial value problem (equation (18)) is based upon certain simplifying assumptions which offer a compromise between reality and mathematical tractability. It has been assumed that: (1) there is a one-to-one correspondence between fuel rate and gas generator speed in the steady-state; (2) the definition of the specific fuel consumption can be extended to serve as a meaningful statement of overall system efficiency for both static and dynamic machine operation; (3) the steady-state operating characteristics of the power turbine can be used as a solution to the problem of determining the accelerating torque on the turbine by formally replacing W_f with h (equation (5)); (4) the two

compressor-turbine pairs change speeds in phase during a transient; and (5) a quasi-static fuel rate can be defined to serve as a measure of the energy needed to sustain the engine at each of the unsettled states that it passes through during a transient.

THE FUEL CONTROLLER

The fuel controller on a gas turbine engine is a multi-input, single-output device. Controller response is characteristically rapid with respect to the response time of the engine. In fact, the dynamic behavior of the fuel controller can be successfully described by assuming that the equations governing its operation do not explicitely depend upon time. Consequently, the time-dependent nature of the fuel controller must be a manifestation of the transient behavior of one or more of its several inputs.

No attempt will be made to analyze the configuration of individual constituent elements in a fuel control system. However, it is possible to construct a mathematical representation of such devices from an analysis of transient data acquired by performing a series of tests designed specifically for this purpose. These tests are based on the assumption that the fuel controller can be regarded as a static nonlinearity of the gas turbine system.

Let W_{fm} and W_{fi} be the maximum and idle (minimum) fuel rates, respectively. Construct the function, W_{fk} defined by the following equation:

$$W_{fk} = W_{fk-1} + \frac{W_{fm} - W_{fi}}{n} \quad \text{lb/hr} \qquad k = 2, \ldots, n \tag{19}$$

where $W_{fo} = 0$ and $W_{f1} = W_{fi}$. At each of the n different steady-state fuel rates defined by equations (19), conduct the following tests on the fuel controller.

At $t = 0^-$, $W_f = W_{fk}$ and $N_h = h^{-1}(W_{fk})$ where h^{-1} is the inverse of the function h defined implicitely by equation (5). Let L be throttle position and thus L_k, L_i, and L_n correspond, respectively, to W_{fk}, W_{fi} and W_{fm}. Introduce a step-change in L from L_k to $L_k + \frac{(L_n - L_k)}{m}$ (m is an integer). Record the final gas generator speed, N_h, and the full transient response of the fuel rate. Reset the throttle to L_k and perform a test similar to the previous one except that in this instance, the throttle is to be advanced to the position $L_k + \frac{2}{m}(L_n - L_k)$. Exhaust the m possible tests in this sequence (L_k to $L_k + \frac{j}{m}(L_n - L_k)$; $j = 1, \ldots, m$) and reset the throttle to L_k. Introduce a throttle step-change from the setting L_k to $L_k - \frac{(L_k - L_{ki})}{m}$. Record the final gas generator speed and the full transient response of the fuel rate. Again, exhaust the m possible tests in this sequence (L_k to $L_k - \frac{j}{m}(L_k - L_i)$; $j = 1, \ldots, m$). Set the throttle to achieve a fuel rate, W_{fk+1} and denote this throttle setting by L_{k+1} and repeat the 2m tests conducted previously. Continue in this manner until all n possible fuel rates (W_{fk}; $k = 1, \ldots, n$) have been explored.

The integer, n, should be greater than or equal to three (3). The integer, m, however, is evidently dependent on the difference $L_n - L_k$ ($L_k - L_i$) for positive (negative) throttle step-changes. Near W_{fm} (W_{fi}) the difference $L_n - L_k$ ($L_k - L_i$) is small and hence the number of tests, m, required in this interval are few (perhaps one). Conversely, large

differences between L_n (L_k) and L_k (L_i) for positive (negative) throttle step-changes may require m to be three or more to ensure that the transient records evince a smooth pattern.

The fuel controller test records for the positive and negative throttle step disturbances are analyzed over the time interval $(0, \epsilon)$ where ϵ is some positive number and zero is the time at which the transient was initiated. Let τ_{fm} be the maximum time constant of the fuel controller. Then ϵ is bounded below by τ_{fm} and above by the requirement that at $t = \epsilon$ gas generator speed, N_h, must not differ significantly from its initial value. Determine the maximum fuel rate in the interval $(0, \epsilon)$ for each transient record. Construct a graph of maximum fuel-rate vs the difference speed, ΔN_h, at each of the n steady-state fuel rates, W_{fk} $(k = 1, \ldots, n)$. The quantity, ΔN_h is determined for each record by the following prescription:

$$\Delta N_h = N_h(\infty) - N_h(0) \qquad \text{rpm} \tag{20}$$

The time-dependent fuel rate can then be expressed mathematically by the following relationship.

$$W_f = B(N_h, \Delta N_h) \qquad \text{lb/hr} \tag{21}$$

Interpret ΔN_h as the error between gas generator speed, N_h, and its speed set point, N_{hs}. Then let E be the function derived from B which takes N_{hs} and N_h into W_f.

$$W_f = E(N_h, N_{hs}) \qquad \text{lb/hr} \tag{22}$$

Denote by D the function which relates L and N_h and replace N_{hs} in equation (22) by D (L) and substitute this result into equation (18) to obtain the following system of equations.

$$\dot{N}_p = \frac{9.54}{J_p} \left(f\left(h(N_h), N_p\right) - Q_1 \right) \qquad \text{rpm/sec}$$

$$\frac{1}{e} J_p N_p \dot{N}_p + \left(J_h N_h + J_1 g'(N_h)\right) \dot{N}_h =$$

$$19.7\gamma \left(E\left(D(L), N_h\right) - C\left(Q_1, N_p\right) \right) \qquad \text{lb/ft} \qquad (23)$$

$$N_p(0) = N_{po} \qquad \text{rpm}$$

$$N_h(0) = N_{ho} \qquad \text{rpm}$$

CONCLUSIONS

A marine propulsion gas turbine engine is dynamically characterisable by a system of two nonlinear differential equations. These equations are a succinct statement of the physical principles underlying the transient behavior of gas turbines in general. Representations of specific systems can be systematically deduced from equation (23) by imposing the appropriate mathematical constraints on these equations. These constraints, in turn, are developed from data derived from: (1) the steady-state characteristics of the engine, and (2) the dynamic responses of the fuel control system to select input disturbances.

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1. Donnelly, J. W., "Analog Computer Simulation of an FT4A-2 Marine Gas Turbine Engine" Report of NAVSECPHILADIV Project C-53-I of 30 June 1971.

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